Solving Multi-Level Problems Using Multi-Objective Optimization for Robust Solutions

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1. Research Project & MRIVCC Theme

➢ **Proposed Bilevel Model**

$$\text{Minimize}_{(\mathbf{x},\mathbf{y})} \quad TotalTransCost = \sum_p \sum_w TransCost_{pw}(\mathbf{x},\mathbf{y}),$$
$$\text{subject to} \quad \mathbf{y} \in \text{argmax}_{(\mathbf{y})} \{TransCost_{pw} = Dist(\mathbf{x}) * Demand * CarrierCost(\mathbf{y})$$
$$Service_{pw} = \sum_c Demand * ServiceQuality(\mathbf{y})\},$$

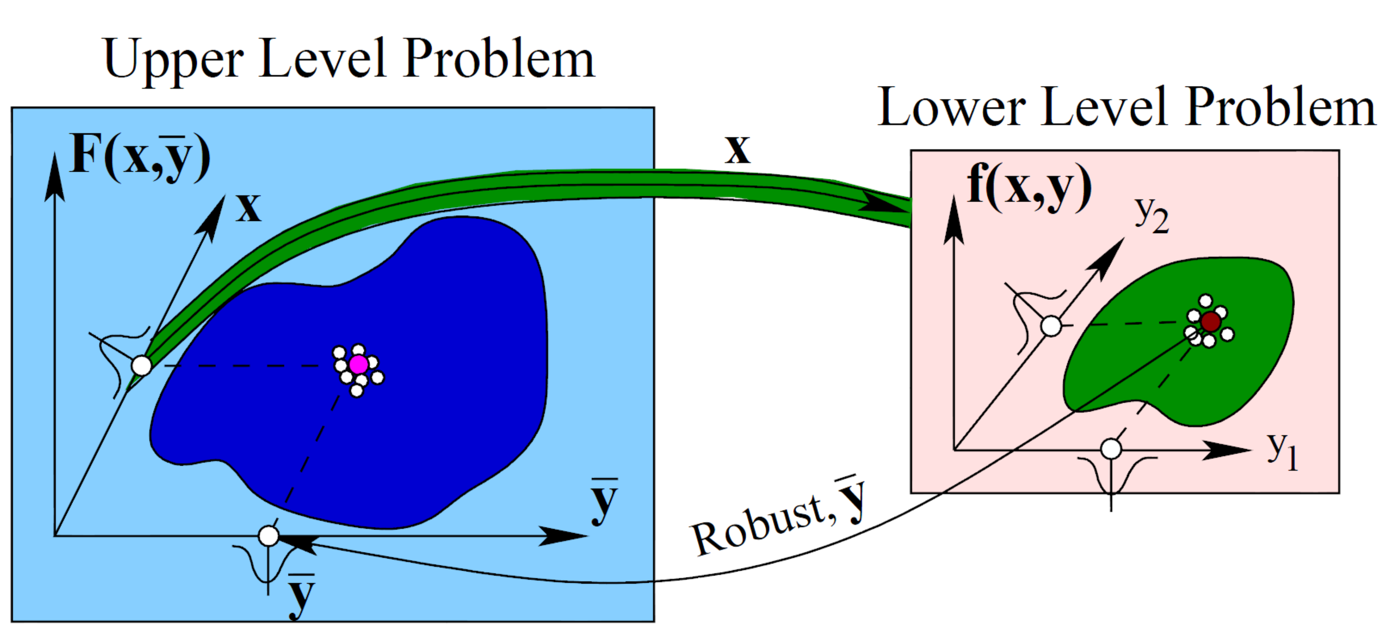
$$\mathbf{x}_i \in [1,10],\ i = 1,2,\ldots,N; \quad N = number\ of\ destinations\ considered.$$
$$\mathbf{y}_j^p \in [1,2,3,4,5],\ j = 1,2,\ldots,n; \quad n = number\ of\ shipments.$$
$$p = 1,\ldots,10 plants; \quad w = 1,\ldots,52 weeks; \quad c = 1,\ldots,5 carriers.$$

➢ **Multiple Levels:**

The supply-chain management involves a hierarchy of sub-problems just like most engineering design and control systems. Not only is such a hierarchical treatment necessary for an accurate evaluation of a process, it also enables a systematic and computationally tractable procedure for performing the overall analysis.

➢ **Multiple Conflicting Objectives:**

Optimization problems in practice most often have multiple conflicting goals arising from different economic, environmental and efficacy-related issues. Hence, a multi-objective optimization method is a ideal way of first finding a set of trade-off Pareto-optimal solutions and then choosing a preferred solution based on multi-criterion decision analysis.



➢ **Uncertainties and Robustness:**

A computational optimization always finds the optimum of the model of the model being used. Practice often is different from the model. The theoretical minimum solution is usually sensitive to decision variables or parameters uncertainties that may arise due to reasons like imperfect implementation. A sub-optimal solution that's robust to uncertainties is always preferred to implement in practice.

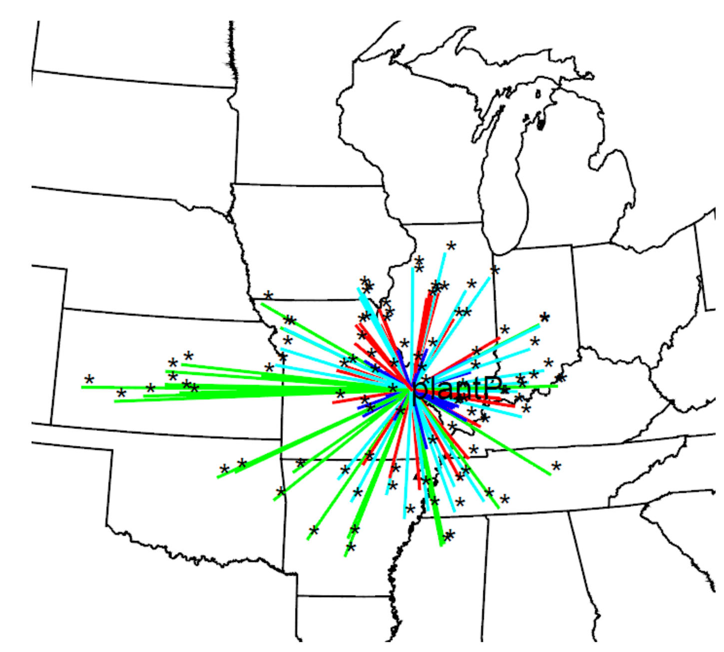
Project theme: Develop a multi-level, uncertainty-based, multi-objective method for optimizing supply-chain management problems.

3. Results/Future Directions

➢ **Demand fulfillment example (lower level solutions)**

➢ **State-level vs. Zipcode-level destinations**

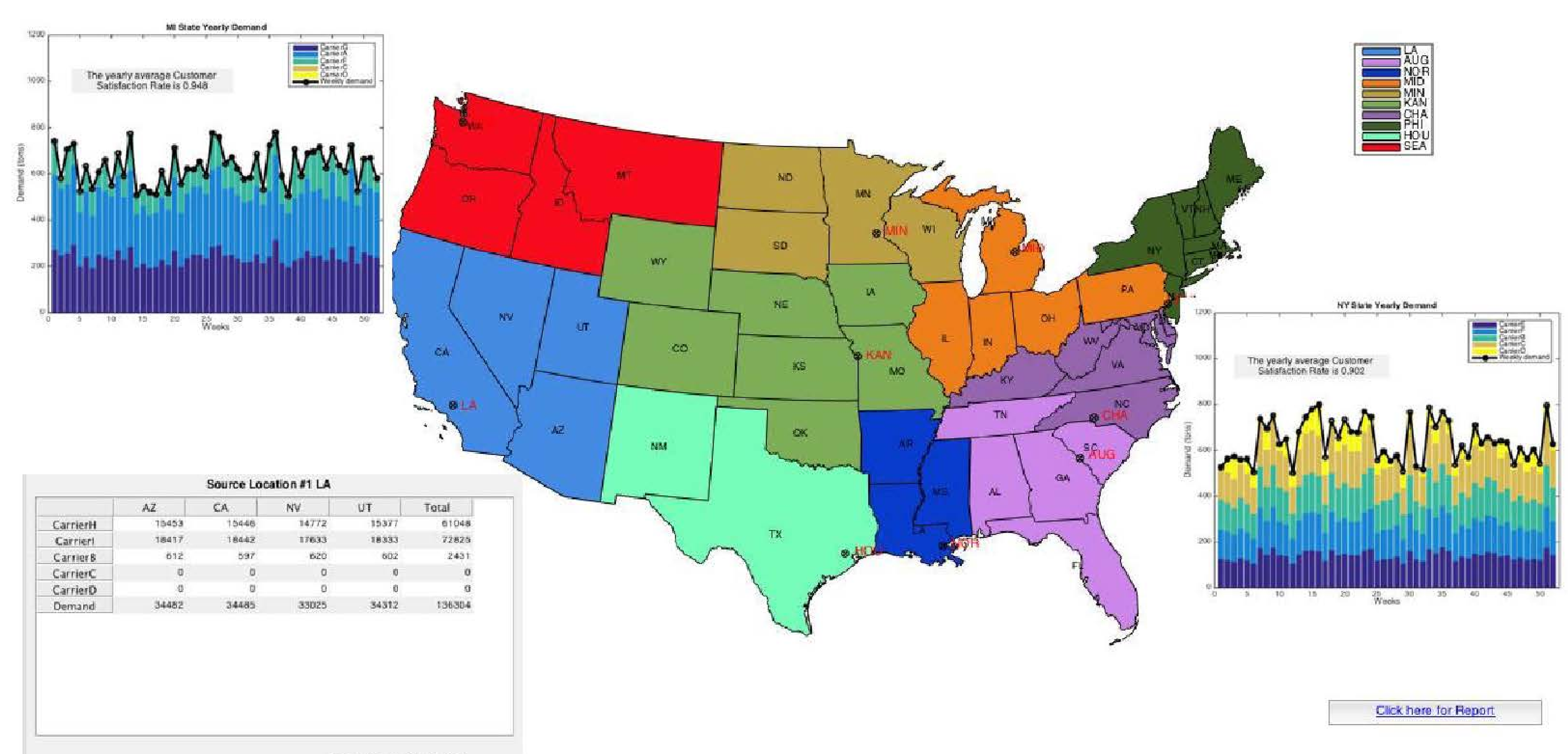
Destination	KY	KS	AR	TN	IN	MO	IL	
Days	2	2	2	2	2	1	1	Capacity
CarrierA	12	0	0	0	0	4	0	28
CarrierB	2	11	2	0	0	5	0	35
CarrierC	0	0	8	9	0	8	0	42
CarrierD	0	0	0	3	14	0	8	42
CarrierE	0	0	0	0	0	0	15	49
Demand	14	11	10	12	14	17	23	



➢ **Results comparison**

Destination	Single-level Simple Averaging(No Optimization)	Bi-level Rule-based	Bi-level Optimization
State-level	\$15.1M	\$12.5M (17%)	\$12.1M (20%)
Zipcode-level	\$14.8M	\$11.7M (21%)	N.A.

➢ **FAT Map (upper level solution)**



➢ **Future direction 1:** Introduce uncertainties to the anticipated demands to reflect the seasonal nature of the problem. Modify the algorithm to find the robust solutions.

➢ **Future direction 2:** Introduce an intermediate level, in which the performance of each carrier company candidate is evaluated. This tri-level problem could be optimized quarterly, and the carrier list (with ordering) found will be then implemented for the next quarter. It is expected to provide a better solution.

2. Value Created

➢ **The proposed approach is expected to produce pragmatic solutions that are economical, robust and implementable.**

➢ **The discovery of multiple trade-off solutions due to multi-objective consideration and uncertainty and uncertainty handling should provide engineers with valuable knowledge and insights about the problem**

➢ **Understanding relationships between strategic, tactical and operational decisions through a multi-level optimization for process and supply-chain optimization.**

The project was initiated in August 2015 and continues until August 2017

4. Project Plan

Milestone 1: Formulate the supply-chain management (facility allocation and transportation assignment) problem into a multi-level optimization problem. Customize a hybrid optimization method combining an evolutionary algorithm and a classical (mixed-integer programming) algorithm to handle zip-code destinations. Post-optimality analysis to exact meaningful rules to avoid frequent optimizations. Completion Date: 1 year from the start of the project.

Milestone 2: Introduce uncertainties to anticipated demands and modify proposed algorithm to obtain robust solutions. Introduce a tri-level formulation of the problem by including an intermediate level. Expected Completion Date: 2 year from the start of the project.